DRAG

We can use the same method to measure drag for a particular AoA as we did for lift. As drag is an aerodynamic component, the formula for zero-lift drag can be expressed as follows:

$$D_0 = C_{D0} \cdot q \cdot S$$

Where C_{DO} is the Coefficient of drag (parasite), **q** is the dynamic pressure and **S** is the surface area. Dynamic pressure is directly proportional to the square of the velocity of the object. Therefore, we can conclude that parasitic drag increases by the square of the speed. The graph below shows this relationship.



Figure 4 - Zero lift drag and speed relationship

The total Coefficient of Drag is composed of two elements, the Skin Friction or Parasitic drag as well as Induced Drag. Induced drag is the drag that is created as a by-product of lift. As the wing moves through the air, a threedimensional flow pattern can be observed. Firstly, the fluid flows outwards from the wing root along the bottom part of the leading edge and spills around the wingtip forming a vortex. Secondly, air flowing over the wing causes an overall downwash of air behind the trailing edge within the span of the wing. This downwash produces a localised flow of air behind the wing, creating a rearward local vector force. As this local vector is inclined backwards, the total lift is decreased and the drag increased.



Figure 5 – Three-dimensional flow

Induced drag is inversely proportional to V^2 and tends to be very high at zero speed (as no lift is produced) and decreases with an increase in velocity. Therefore, at low speeds induced drag is predominant whilst D_0 is very small.



Figure 6 - Induced Drag curve

By transposing both graphs we obtain a total drag curve similar to the one below. It can be seen that where the value of $D_i = D_0$ meet, we obtain the speed for minimum drag called V_{MD} . This speed is very important as is the most efficient airspeed (i.e.: where the compromise in parasitic and induced drag is at its minimum). The total drag at any point along both curves is the sum of the Induced and Parasitic drag ($D_i + D_0 = D_{TOT}$).



Figure 7 - Total Drag curve

COEFFICIENT OF DRAG

The Coefficient of Drag (C_D) is part of the total drag equation and takes into account the surface area of the wing (S), shape of the aircraft, wing planform and Aspect Ratio (AR). The total C_D is composed of two elements of drag (induced and parasitic) and can be expressed as follows:

$$\mathsf{D}_{\mathsf{TOT}} = \frac{1}{2}\rho \cdot \mathsf{V}^2 \cdot \mathsf{S} \cdot \mathsf{C}_\mathsf{D}$$

We can solve for the Coefficient of Drag by re-arranging the equation:

$$C_{D} = \frac{D_{TOT}}{q \cdot S}$$

We can express C_D as the proportion of the total drag force to the force produced by the dynamic pressure per area. One element of this equation that is not a variable is the surface area of the wing (which does not change). The dynamic pressure has a direct impact on the C_D – for example, at cruising speed where we have relatively low angles of attack, the C_D is low but the airspeed (V) is high, which gives a higher total drag. Conversely, at very high angles of attack (close to stalling), we know that the airspeed is quite low and the drag is very high.

Therefore, the optimum angle of attack corresponds to the most efficient airspeed for minimum drag (V_{MD}) as there is a direct relationship between the dynamic pressure and the total drag. Also, an increase in aircraft mass will have a direct impact on the total drag – due to the fact that a higher lift (and concurrently, more drag) is required to counteract the weight. *Remember that induced drag is a by-product of lift*!

It can be said that as the weight increases, the optimum speed also increases. This can be said for any flight manoeuvres or aircraft configuration changes (landing, takeoff, etc.).

For example, if we were required to perform a turn at a constant altitude, we require an excess of lift in order to keep the aircraft at the same level - resulting in a slight back-pressure on the control column (or stick) during the turn, in order to avoid descending.

POLAR AND LIFT/DRAG RATIO CURVES

During aircraft design, we must take both the elements of lift and drag to determine the performance and efficiency of an aerofoil at different angles of attack and airspeed. We can plot these elements on a Polar Curve, which gives the variation of C_D as a function of C_L for each Angle of Attack.

On the graph on the right, we divide C_L by C_D to obtain a curve showing the Lift to Drag ratio (L/D Ratio) as a function of a particular angle of attack. The graph on the left is a polar curve and the one on the right is the L/D ratio to AoA.





Figure 8 - Polar curve and L/D Ratio to AoA

From the polar curve, we can determine the different characteristics of the wing. For example, if we draw a line that is parallel to the vertical axis (C_L) and meets the curve with the lowest value of C_D , we can find the angle corresponding to the least resistance.

Tracing a line parallel to the horizontal axis and intersecting the curve at the top will give us the angle of attack with the maximum value of C_L .

The best relationship between C_L and C_D will be the point originating from the origin of both axes to the tangent of the curve. This will give us the Maximum Efficiency of the aerofoil. And in this case, will give us the angle of incidence which will give us the greatest range (best relationship between lift and drag). This is an important value as it is the most efficient (also in terms of fuel consumption for an aircraft):

Efficiency (E) =
$$C_L / C_D$$

For example, by intersecting the tangent on the curve originating from the origin of both axes, we find a corresponding value for C_L and C_D at best efficiency. This relationship also gives us the best ratio of lift to drag.

Example:

We draw a line from the origin of the polar curve that intersects the tangent of the curve, at best efficiency which give us the following values of $C_L = 0.6$ and $C_D = 0.046$.

Therefore, the best lift/drag ratio at (E) = 0.6 / 0.046 = 13

This means that lift is 13 times the drag produced. Normal aircraft have a range of typically 12 to 20 and gliders can have an efficiency between 28 and 61!

In fact, the force needed to lift a glider weighing 500 kg can be as little as 100 N or just 10 kg! Due to the high L/D ratio.